64/100

Examination Book

NAME Refath

SUBJECT

INSTRUCTOR

EXAM SEAT NO. SECTION

DATE GRADE

1. 21
2. 12
3. 22
4. 9

64

1a) $\vec{E}(\vec{r},t) = (-25\hat{\imath} - 25\hat{\jmath} + 50\hat{k})\frac{V}{m}$ *

$\cos\{5\pi\times10^{6}(x+y+z) - \underbrace{10^{15}\pi}_{\omega} t\}$

$v = \frac{\omega}{k}$ where $\vec{E} = E_0 \cos(\vec{k}\cdot\vec{r} - \omega t)$

$\omega = 10^{15}\pi$ $\quad \vec{k}\cdot\vec{r} = k_x x + k_y y + k_z z$

$\therefore k_x = k_y = k_z = 5\pi\times10^{6}$

$v = \frac{10^{15}\cancel{\pi}}{5\cancel{\pi}\cdot10^{6}\sqrt{3}}$

$k = \sqrt{k_x^2 + k_y^2 + k_z^2}$

$= \sqrt{(5\pi\times10^{6})^2 \cdot 3}$

$= 5\pi\times10^{6}\sqrt{3}$ ✓

unit ??

$v = \frac{10^{9}}{5\sqrt{3}}$

$\boxed{v \approx 0.115\times10^{9}\ m/A}$ } Magnitude ✓

$\hat{v} = \hat{k} = \hat{\imath} + \hat{\jmath} + \hat{k} = (1,1,1) = \boxed{\frac{1}{\sqrt{3}}\hat{\imath} + \frac{1}{\sqrt{3}}\hat{\jmath} + \frac{1}{\sqrt{3}}\hat{k}}$ } Direc.

moving rightwards

$n = \frac{c}{v} = \frac{3\times10^{8}}{0.115\times10^{9}} = \boxed{26.08\times10^{-1} \approx 2.6}$ ✓

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1b) $\vec{E}_0 = c\vec{B}_0 \rightarrow B_0 = \frac{E_0}{c} = \frac{-25\hat{\imath} - 25\hat{\jmath} + 50\hat{k}}{3\times108}$ ✗

$\rightarrow v$

$B = \frac{-25}{3\cdot10^{8}}(\hat{\imath} + \hat{\jmath} - 2\hat{k})\,T \cos\{5\pi\times10^{6}(x+y+z) - 10^{15}\pi t\}$

tesla

in-phase with $\vec{E}(r,t)$, assuming linear polarization

$$\vec{B} = \frac{1}{\omega}\vec{k}\times\vec{E}$$

$$\vec{B} = \frac{1}{10^{15}\pi}\left(\begin{array}{c|ccc|} & \hat{i} & \hat{j} & \hat{k} \\ k= & 1 & 1 & 1 \\ E_0= & 1 & 1 & -2 \end{array}\right) \begin{array}{l} \hat{i}(-2-1) \quad -3\hat{i} \\ -\hat{j}(-2-1) = +3\hat{j} \\ +\hat{k}(1-1) \quad +0\hat{k} \end{array}$$

$$\vec{k} = (5\pi\times10^6, 5\pi\times10^6, 5\pi\times10^6) = 5\pi\times10^6(1,1,1)$$

$$\vec{k}\times\vec{E} = 5\pi\times10^6(\hat{i}+\hat{j}+\hat{k})\times(-25\hat{i}-25\hat{j}+50\hat{k})$$
$$= (-25)(5\pi\times10^6)(\hat{i}+\hat{j}+\hat{k})\times(\hat{i}+\hat{j}-2\hat{k})$$

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$$\vec{B} = \frac{1}{10^{15}\cancel{\pi}}(-25)(5\cancel{\pi}\times10^6)(-3\hat{i}+3\hat{j}+0\hat{k})$$

$$\boxed{\vec{B} = -125\times10^{-9}(-3\hat{i}+3\hat{j}+0\hat{k})\cdot\cos\{5\pi\times10^6(x+y+z)-10^{15}\pi t\}}$$

unit??

Check: $E\cdot B = (\hat{i}+\hat{j}-2\hat{k})\cdot(-3\hat{i}+3\hat{j}+0\hat{k})$
$= -3+3+0 = 0$ ✓

1c) $\mathcal{P} = \frac{I}{c}$ for absorption

$\mathcal{P} = \frac{2I}{c}$ for reflection

$\mathcal{P} = 0.4\frac{I}{c} + 0.6\frac{2I}{c}$

$\frac{I}{c} = \frac{W/m^2}{m/A}$

$\frac{W}{m^2} \cdot \frac{A}{m}$

$\frac{WA}{m^3}$

$\frac{F}{A} = \frac{N}{m^2}$

$I = \frac{EB}{2\mu} = \frac{E^2}{2\mu c} = \frac{(25\sqrt{6})^2 \frac{V^2}{m^2}}{2(4\pi \times 10^{-7} \frac{mkg}{C^2})(3\times10^8 \frac{m}{s})}$

$E = \sqrt{25^2+25^2+50^2} = \sqrt{a^2+a^2+(2a)^2} = \sqrt{6a^2}$
$\underset{a}{25}$ $= a\sqrt{6} = 25\sqrt{6}$

$\mu \approx \mu_0$ for dielectrics

$I = \frac{25^2 \cdot 6}{24\pi \times 10}\frac{W}{m^2} = \frac{3750}{240\pi} \approx 4.97\frac{W}{m^2}$

$\mathcal{P} = 0.4\frac{4.9\, W/m^2}{3\cdot10^8\, m/A} + 0.6\frac{2(4.9)\, W/m^2}{3\cdot10^8\, m/A}$ ✓ ✓

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$\mathcal{P} = 0.4\times1.633\times10^{-8} + 0.6\times3.266\times10^{-8}$

$\mathcal{P} = 10^{-8}(0.6533+1.9596)$

$\boxed{\mathcal{P} = 2.6129\times10^{-8}\, N/m^2}$ X

Hooo!!

2a) A_0 acts as a "source" of coherent waves. It is presumed that S_0 is placed sufficiently far from the wave source so that the wavefronts are approximately planar once they reach S_0. ?!?

Sphereical!! The symmetric placement of S_1 and S_2 are neccessary because otherwise, the two waves would not be in-phase when exiting the double slits. Instead, there will be a path difference because one wave had to travel less (if the slits were not symmetric)

What makes them in phase??

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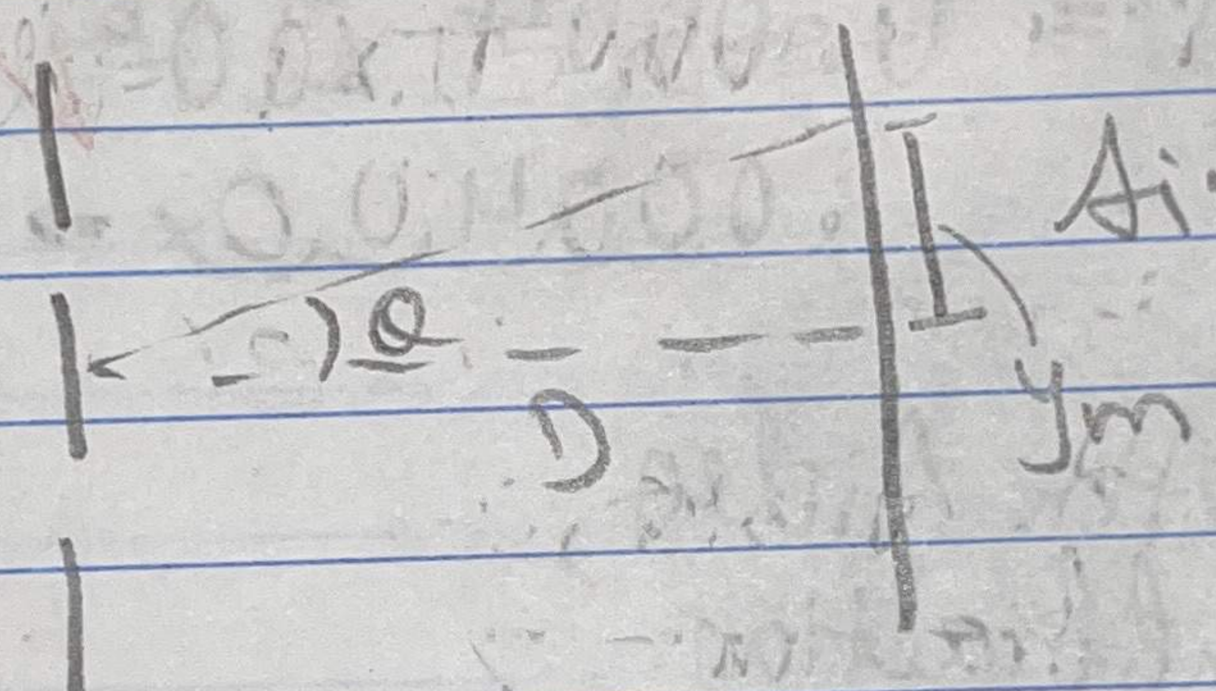
2b) Diffraction min: $a\sin\theta = m\lambda$

Diffraction max: $a\sin\theta = (m+\frac{1}{2})\lambda$

$m = 2$ for second minima:

$\lambda = 590\,nm$ $D = 1.7\,m$

$a = 0.25\,mm$ $d = 0.75\,mm$



$\sin\theta \approx \tan\theta = \frac{y_m}{D}$

$D\tan\theta = y_m$

$a\sin\theta = 2\lambda \rightarrow \tan\theta = \frac{2\lambda}{a}$

$y_m = D\left(\frac{2\lambda}{a}\right) = \frac{2\lambda D}{a}$

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$$y_m = \frac{2(590\times10^{-9}\,m)(1.7\,m)}{(0.25\times10^{-3}\,m)}$$

$$y_m = \frac{2006\times10^{-9}\,m^2}{0.25\times10^{-3}\,m} = 8024\times10^{-6}\,m$$

$= 0.008024\,m$

$= .8\times10^{-2}\,cm$

$\therefore 2y_m = 2(.8\times10^{-2}\,(cm))$

$= \boxed{1.6\,cm}$

how many diffraction minima?

$\theta = 90° \rightarrow a \sin\theta = m\lambda$

$a = m\lambda \rightarrow m = \frac{a}{\lambda}$

$m = \frac{0.25 mm}{590 nm} = \frac{0.25 \times 10^{-3} m}{590 \times 10^{-9} m}$

$m = 0.0004 \times 10^{6}$ ↳ How

$.000400. \Rightarrow m = 400$ diffraction minima on ONE side

for both sides,
diffraction minima $= 2 * 400 = \boxed{800}$ ~~X~~

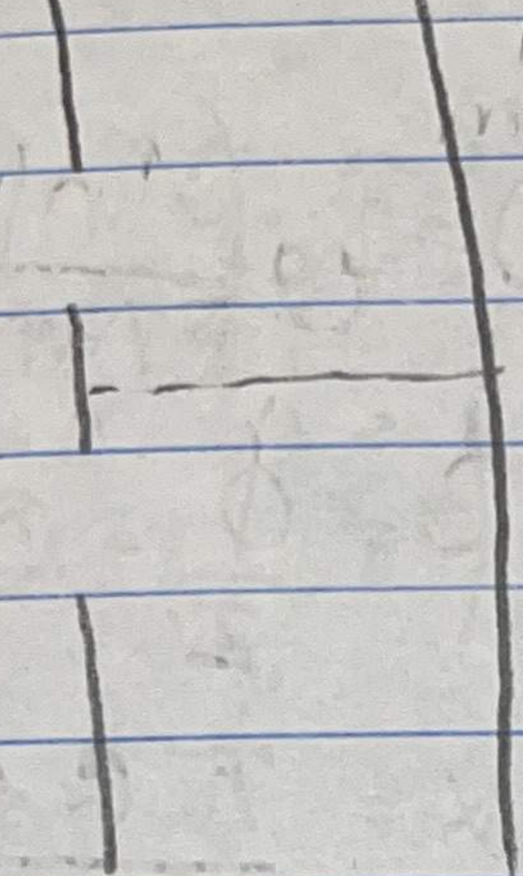
2c) # interference maxima w/i central diffraction maxima

$d\sin\theta = m\lambda$

$a\sin\theta = (m + \frac{1}{2})\lambda \quad m = 0, 1, 2, \ldots$

central

$a \sin\theta = \frac{1}{2}\lambda \rightarrow \sin\theta = \frac{\lambda}{2a}$



<u>Plan</u>: → find angular width θ_M of central diffraction maxima

→ plug in different m's in $\sin\theta = \frac{m\lambda}{d}$ until $\theta > \theta_M$

$$\sin\theta = \frac{\lambda}{2a} = \frac{590\times10^{-9}m}{2(.25\times10^{-3}m)}$$

$$= \frac{590\times10^{-9}m}{0.5\times10^{-3}m}$$

$$= 1180\times10^{-6}m$$

$$\sin\theta = 1.180\times10^{-3}m$$

$$\sin\theta = m\,\frac{590\times10^{-9}m}{0.75\times10^{-3}m} = 786.6\times10^{-6}m$$

$$= (7.866\times10^{-4}m)$$

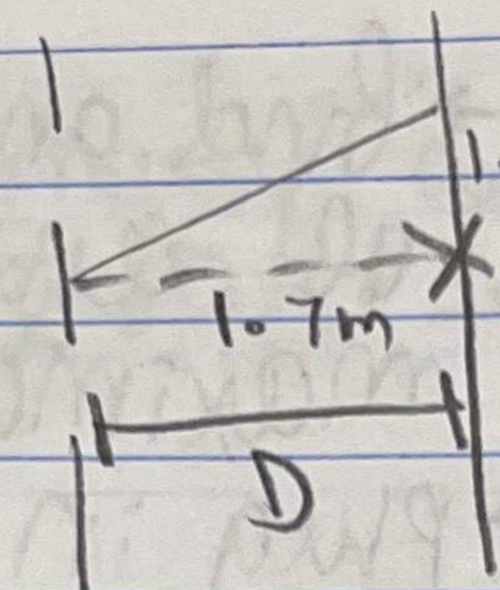
m

0.7866

$$\frac{1.18}{0.7866} = \boxed{1.5 \text{ maxima}}$$

or 1 maxima

2.

2d) 

$I(\theta=0)=I_0\left(\frac{\sin N\beta}{\sin\beta}\right)^2\left(\frac{\sin\alpha}{\alpha}\right)^2$

where $\beta=\frac{\phi}{2}=\frac{\pi d\sin\theta}{\lambda}$

$\lambda=590\,nm$

$a=0.25\,mm,\ D=1.7\,m$ $\quad \alpha=\frac{\pi a\sin\theta}{\lambda}$

$d=0.75\,mm$

$\sin\theta\approx\tan\theta=\frac{y_m}{D}=\frac{1.7\,mm}{1.7\,m}=\frac{1.7\times10^{-3}m}{1.7\,m}$

$\sin\theta\approx\tan\theta=10^{-3}\rightarrow\beta=\frac{\pi d\cdot10^{-3}}{\lambda}$

$\beta=\frac{\pi(0.75\times10^{-3}m)\cdot10^{-3}}{590\cdot10^{-9}m}$ $\quad \alpha=\frac{\pi a\cdot10^{-3}}{\lambda}$

$=\frac{.75\pi\cdot10^{-6}m}{590\cdot10^{-9}m}=\frac{2.35}{590}\cdot10^{3}$

$\beta=0.00399\cdot10^{3}$

$\beta\approx4°$

$\alpha=\frac{\pi(0.25\times10^{-3}m)\cdot10^{-3}}{590\cdot10^{-9}m}=\frac{0.25\pi\times10^{-6}m}{590\times10^{-9}m}$

$\alpha=0.0013\times10^{3}\approx1.3°,\ N=2$

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Need iteration !!

$$I(\theta = \tan^{-1}(10^{-3})) = 10\left(\frac{\sin(2\cdot 4)}{\sin(4)}\right)^2\left(\frac{\sin(1\cdot 3)}{1\cdot 3}\right)^2$$

$$I = 10\left(\frac{0.13}{0.069}\right)^2\left(\frac{0.022}{1.3}\right)^2$$

$$I = 10(1.88)^2(0.0169)^2$$

$$I = 10(3.53)(0.00028)$$

$$I = 10(0.001) = 0.01\ \mathrm{W/m^2}$$

$$\boxed{I = 0.01\ \mathrm{W/m^2}} \quad // \quad \times$$

3a) $E(z,t) = 8.0\ \frac{kV}{m}\,\hat{i}\sin(kz - \omega t) \Rightarrow \omega t = \frac{2\pi}{\cancel{T}}\,\frac{\cancel{T}}{8}$

$+\ 6.0\ \frac{kV}{m}\,\hat{j}\sin(kz - \omega t - \frac{\pi}{4}) \quad \frac{2\pi}{\cancel{T}}\,\frac{\cancel{T}}{4}$

Amplitudes are not equal and phase Shift $\phi = \frac{\pi}{4} \Rightarrow$ ELLIPTICAL POLARIZED ✓

$E(0,0) = 8\ \frac{kV}{m}\,\hat{i}\sin(0) + 6\ \frac{kV}{m}\,\hat{j}\sin(-\frac{\pi}{4})$ ✗

Left Elliptical Polarized (Clockwise)

$E(0,0) = -\frac{6\sqrt{2}}{2}\ \frac{kV}{m}\,\hat{j}$

$E(0,\tau/8) = 8\ kV/m\,\hat{i}\sin(-\frac{\pi}{4}) + 6\ \frac{kV}{m}\,\hat{j}\sin(-\frac{2\pi}{8})$

$= -\frac{8\sqrt{2}}{2}\ \frac{kV}{m}\,\hat{i} - 6\ \frac{kV}{m}\,\hat{j}$ ✓ How??

$E(0,\tau/4) = 8\ kV/m\,\hat{i}\sin(-\frac{\pi}{2}) + \cancel{6\ \frac{kV}{m}\,\hat{j}\sin(-\pi)} \rightarrow 0$

$= -8\ kV/m\,\hat{i}$

3a) Orientation of ~~Left~~ Elliptically Polarized Wave:

$$\tan(2\alpha) = \frac{2 E_{ox} E_{oy}}{E_{ox}^2 - E_{oy}^2} \cos \varepsilon$$

$E_{ox} = 8 \, kV/m \quad E_{oy} = 6 \, kV/m$

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$$\tan(2\alpha) = \frac{2(8\,kV/m)(6\,kV/m)}{(8\,\frac{kV}{m})^2 - (6\,kV/m)^2} \cos(\frac{\pi}{4})$$

$$= \frac{48\,\cancel{96}\,kV^2/m^2}{28\,kV^2/m^2} \frac{\sqrt{2}}{\cancel{2}}$$

$$= \frac{48\sqrt{2}}{28} = \frac{12\sqrt{2}}{7} = 2.42$$

$$2\alpha = \tan^{-1}(2.42) = 67.58$$

$$\boxed{\therefore \alpha = 33.79^\circ \text{ from } +x \text{ axis}}$$

$I = I_0 \cos^2(\theta)$

$\theta = \alpha = 33.79$ & $I \propto |E|^2$

?? $I_0 = |E_0|^2 = \left(\sqrt{6^2+8^2}\right)^2 = 10 \text{ W/m}^2$ How

$I = 10\cos^2(33.79°)$

$I = 10(0.831)^2 = 10(0.69)$

3

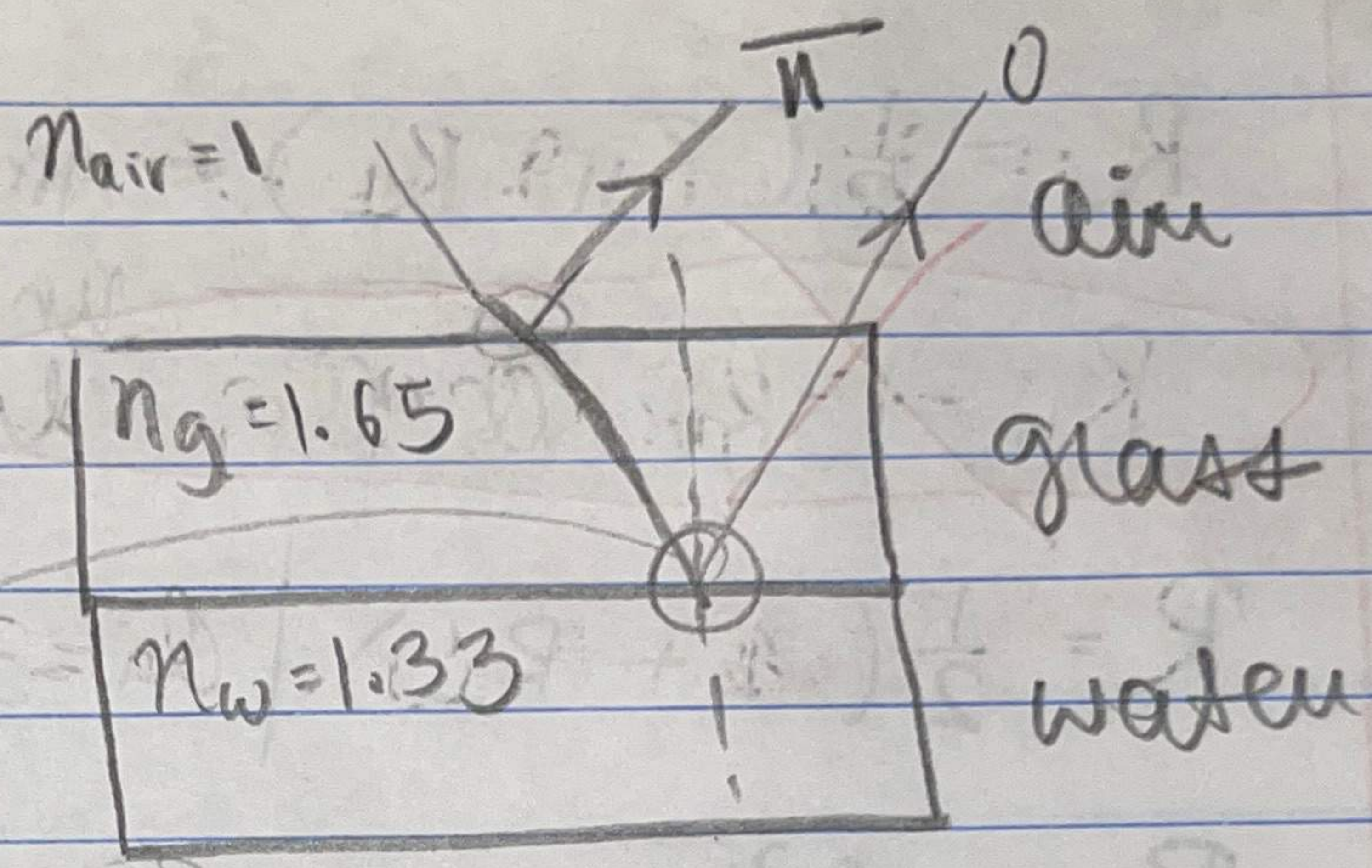
$\boxed{I = 6.9 \text{ W/m}^2 \text{ after polarizer}}$

$\boxed{E_{new} = 6.9 (\text{kV/m}) \hat{i} \sin(kz - \omega t)}$

only component of E in x-direction survives polarizer

So $\hat{i}(\ldots)$ is what you should look for ??

3b)



if $n_t > n_i \Rightarrow \pi$ - shift (HARD REFLECTION)

net phase shift : π

$\theta_p = \tan^{-1}\left(\frac{n_t}{n_i}\right) = \tan^{-1}\left(\frac{1.33}{1.65}\right)$

$\theta_p \approx 38.8°$

$n_i \sin\theta_i = n_t \sin\theta_t$

3 $1.65 \sin 38.8 = 1.33 \sin\theta_t$

$1.033 = 1.33 \sin\theta_t$

$.77 = \sin\theta_t \Rightarrow \theta_t = 51°$

$\boxed{\theta_t = 90 - \theta_p = 51.2°}$

$R = \frac{1}{2}(R_{\parallel} + R_{\perp})$ ← since natural un-polarized light

$\cancel{R_{\parallel} = 1 \text{ for } \theta = \theta_p}$

$R = \frac{1}{2}(1 + R_{\perp})$ | $\theta_i = 38.8, \theta_t = 51.2$

$R_{\perp} = r_{\perp}^2 \Rightarrow r_{\perp} = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)}$

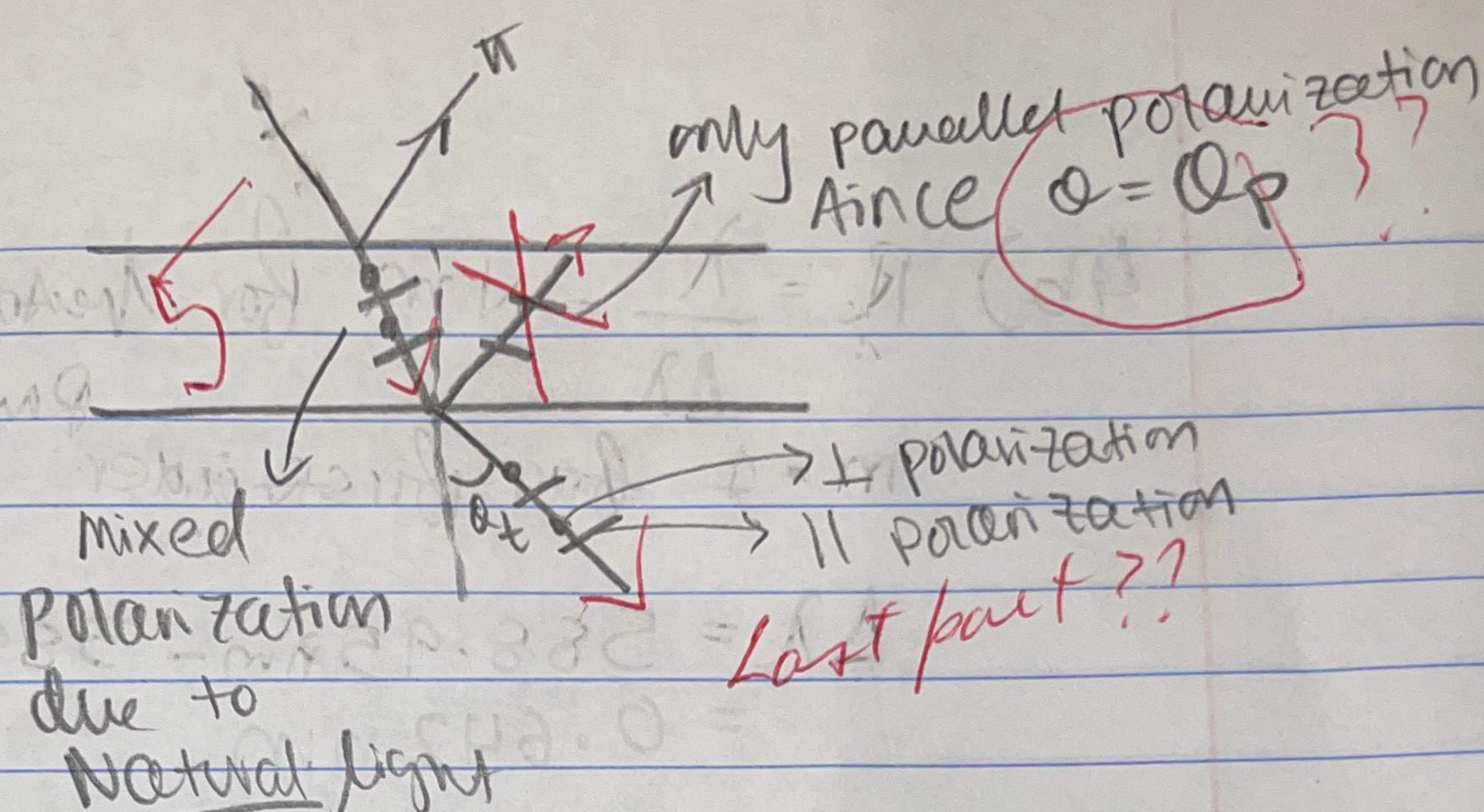
3

$r_{\perp} = -\frac{\sin(38.8 - 51.2)}{\sin(38.8 + 51.2)} = 0.2147$

$\therefore R_{\perp} = r_{\perp}^2 = (0.2147)^2 = 0.046$

$R = \frac{1}{2}(1 + 0.046) = 0.523$

$\boxed{R = 0.523}$



4a) $n = 1.434$ $\qquad \underbrace{\Lambda}_{\text{path difference}} = (n-1)L \rightarrow \Lambda = \frac{(\Delta m)\lambda}{2}$

$\lambda = 589\,nm$

$\Delta m = 35$

$L = ?$

$\rightarrow$ Amplitude splitting

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$(1.434 - 1)L = \dfrac{35 * 589 * 10^{-9}\,m}{2}$

$L = \dfrac{35 * 589 * 10^{-9}\,m}{2(1.434-1)}$

$L = \dfrac{20615 * 10^{-9}\,m}{0.868}$

$L = 23{,}750 * 10^{-9}\,m$

$\boxed{L = 2.375 * 10^{-5}\,m}$ thickness

4b) $R = \frac{\lambda}{\Delta\lambda} = Nm$ for resolving power

$m = 1$ for first-order

$\Delta\lambda = 588.95\,nm - 589.592$
$= 0.642\,nm$

3

$N = \frac{\lambda}{m\Delta\lambda} = \frac{\lambda}{\Delta\lambda} = \frac{588.95}{0.642}$

$\boxed{N \approx 917 \text{ gratings/lines}}$

Rest??